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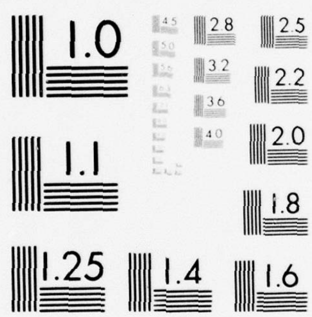

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ATTITUDE DETERMINATION TECHNIQUES FOR A FLIGHT CONTROLLED SCANNING ROCKET

By

Brian F. Sullivan
Marvin E. Stick
Paul E. Connolly

SPACE DATA ANALYSIS LABORATORY
BOSTON COLLEGE
Chestnut Hill, Massachusetts 02167

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Attitude Determination	Rate Integrating Gyro	
Tangent Height	Kalman Filter	

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Mathematical and data analysis techniques are developed to determine the attitude and attitude dependent outputs for the proposed flight of the Aerobee Rocket A35.191-4. The on-board attitude measuring systems consisting of a star mapper, rate integrating gyro, solar aspect sensor, and measurements from a gyroscopic platform will be used in varying combinations to properly determine the orientation of the vehicle during its flight. The attitude accuracy (Cont.)

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20. Abstract (Cont.)

requirements during the special phases of flight (ELE and sawtooth zodiacal scans) dictate the combination of sensors to be used. In addition, final attitude data will be combined with the ephemeris data to calculate tangent height during the ELE scan.

PREFACE

We wish to extend our appreciation to Mr. Leo F. Power, Jr., the Director of the Laboratory, for his continual supervision and recommendations during the preparation of this report. Acknowledgements are also extended to the support staff of this laboratory for their continual assistance in the programming preparation, and in particular to Miss Mary L. Kelly for her efforts and understanding while typing the manuscript.

For his invaluable assistance during the development of the analyses described herein, we express our heartfelt thanks to Mr. Robert E. McInerney of the Air Force Geophysics Laboratory (AFGL).

B.F.S., M.E.S., P.E.C.

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1. INTRODUCTION

The mathematical analysis and data reduction techniques described in this report were developed in conjunction with our role as data support personnel for the proposed rocket flight A35.191-4 whose flight profile has two major phases during which experimental measurements will be taken. Our principal participation in this project is to provide necessary programming and analytic support to determine the attitude and other requested information, using chiefly, the on-board attitude measuring systems.

The primary experimental mission of this proposed flight is the Earth Limb Experiment (ELE), which requires extremely accurate attitude information. To attain the necessary attitude accuracy for the ELE, the attitude measuring system for this portion of the flight consists of an Adcole High Resolution Digital Solar Aspect System Model 15380-1, a Star Mapper, and Honeywell Rate Integrating Gyro. Transformations of coordinate systems, analysis, and data reduction were developed which would be used to calculate the vehicle's attitude from the on-board attitude sensing devices. The attitude will then be used to determine tangent height.

After the ELE scans have been completed, the solar aspect sensor is re-orientated and the vehicle itself torqued to a new predetermined orientation to start the zodiacal scan. The precise knowledge of this orientation, however, does not meet the requested attitude accuracy requirements. Accurate knowledge of the orientation of the vehicle for the start of the zodiacal scan is essential since the initial attitude for the zodiacal scan will be calculated using gyroscopic and Rate Integrating Gyro (RIG) outputs whose zero reference will be this new orientation.

The attitude for the initial point at the start of the scan will be calculated from the preselected orientation of the rocket, updated by correction factors established during the ELE scans. Using this initial estimate, sun sensor and gyroscopic data available in a neighborhood of this initial point, the Kalman filter will be used to update this initial point after the initial point has been determined, attitude during the scan will be computed using gyro

and RIG information. Similar to the ELE portion of the flight, the Kalman filter will be used in sequential fashion to calculate attitude updates based on the sun sensor data.

Digital tapes and printouts containing updated attitude and requested calculations based on attitude information along with an estimate of the possible error in the data will be supplied as final outputs of this project.

2. FUNCTIONAL DESCRIPTION FOR ATTITUDE

To provide extremely accurate attitude information, not only during the ELE operational periods but also during other portions of vehicle lifetime, six reference coordinate systems must be introduced. These systems are required because of the vehicle's flight profile and on-board attitude measuring systems. A discussion of the attitude measuring systems, the flight profile and the reference coordinate systems follows.

2.1 Attitude Sensing Systems

The primary attitude determination system is comprised of a star mapper, sun sensor and Rate Integrating Gyro (RIG). This system will provide necessary attitude information during the ELE operational period. The secondary attitude determination system is an Attitude Control System (ACS) which is comprised of a Roll Stabilized Platform (RSP) with two free gyros, which measure vehicle yaw, pitch, and true roll and three rate gyros. The ACS system is used during the transition phase to maneuver the vehicle to its predetermined orientation so that all experiments will point in their required directions.

2.2 Flight Profile

The A35.191-4 vehicle should behave like a normal well-behaved rocket until 64 seconds after launch when the ACS is activated. After receiving a start signal, the ACS initiates yo-yo despin and payload-vehicle separation. After separation the payload roll position is brought into alignment with the roll gyro reference (roll capture). While roll capture is in progress, pitch and yaw rates are brought to zero under control of the rate gyros. Payload pitch and yaw attitude are aligned with the gyro references following roll capture (three-axis capture). A set of "remote adjust" maneuvers in roll, pitch, and yaw is performed following initial alignment of the payload with the gyros. Upon completion of the remote adjust, the payload sensor axis is aligned to the preprogrammed orientation from which the first of the ELE scans begins. It is to be noted that the average error due to the torquing on the yaw and pitch axes is $\approx 1.6°$ as established from previous Hi-Star flights.

The telemetered attitude data will be corrected, when possible, for systematic errors inherent to the sensor measuring system. These errors will be determined from the study of the various attitude sensors' specifications, i.e. gyro bias drift. The angle between the sun line vector and the center line of

the sun sensor will be predicted and compared with actual sun sensor angular readout. The attitude will be updated, when necessary, using the results of this comparison. Further, the attitude determination during the ELE scans makes it possible to readjust the preselected orientations of the rocket given at the start of a scan and could possibly provide error correction factors which would enable additional preselected orientations to be corrected.

There will be four vertical ELE scans, each in a Northerly direction, in a manner such that Polaris will be detected by the star mapper on each scan. The attitude of the rocket's scanning axis will be determined during each scan by combining the discrete measurements of the star mapper and solar aspect sensor and integrating them with the RIG rate of change information for this axis. The attitude analysis has been developed assuming that all ELE scans will be in a vertical plane. Any deviations or changes will cause modifications in our analysis.

After all ELE scans are completed, the sun sensor will be re-oriented and the vehicle torqued to a new preselected orientation so that a scan (zodiacal) for another experiment can be performed. The ACS, comprised of the RSP with two free gyros, three course rate gyros (one on each of the measuring axes) and two accurate rate integrating gyros (one on the pitch axis, the other on the yaw), will be used in conjunction with the outputs from the sun sensor to determine the most accurate attitude possible during the zodiacal scan. Since the gyros as well as the vehicle are torqued to a new orientation at the start of the zodiacal scan, thus determining a new gyro zero reference, this orientation must be known as accurately as possible. The preselected orientation of the rocket will be used at the start of this scan as the attitude of the rocket, and will be updated by correction factors established during the ELE scans.

Once the best estimate of this orientation has been determined, the attitude will be calculated by standard attitude analysis for gyroscopic data. However, the data reduction analysis of the gyro data will have to be modified, since during this scan the vehicle pitches outside the calibrated limits of the instrument. This vehicle motion causes the output of the pitch signal to be saturated and to remain so while the vehicle is pitching outside the pitch axis calibration ($\pm 12^\circ$). This problem will be corrected by utilizing the outputs of the pitch RIG to produce continuous angular pitch values during this saturated period. This technique entails using the pitch angular value before it is saturated and adding to it the output of the pitch RIG, i.e.,

$$\text{pitch}(t_{k+1}) = \text{pitch}(t_k) + \text{pitch RIG}(t_{k+1}) + E_\sigma$$

where E_σ is the error estimate.

Using this approach in areas in which the pitch angular output is not saturated, we will compare results and hence determine the error estimate. This data reduction technique will then allow us to use the gyroscopic attitude determination analysis mentioned above.

To update the attitude during the zodiacal scan, the sun sensor output will be used. This procedure, similar to that mentioned for the ELE, involves a comparison of the predicted and measured angle between the sun line vector and orientation of the sun sensor axis.

If we let P be the orientation of the sun sensor probe axis, given in the fixed inertial system (I J K) by

$$\hat{P} = I \cos\theta_p \cos\phi_p + J \cos\theta_p \sin\phi_p + K \sin\theta_p$$

and the unit sun line vector as derived from empirical sources given by

$$\hat{S} = I \cos\theta_s \cos\phi_s + J \cos\theta_s \sin\phi_s + K \sin\theta_s$$

where

θ_s is the declination of the sun

ϕ_s is the right ascension of the sun,

then ψ, the angle between $\hat{P}$ and $\hat{S}$ is

$$\psi = \cos^{-1}(\hat{P} \cdot \hat{S}) = \cos\theta_s \cos\theta \cos(\phi - \phi_s) + \sin\theta \sin\theta_s .$$

This predicted value is compared with the measured sun sensor output. Then the attitude, if needed, will be adjusted within an established error bar to achieve the best possible comparison.

2.3 Reference Systems

2.3.1 Fixed Inertial System

The most basic of the six reference coordinate systems to be used is the fixed inertial system (I J K). I is a unit vector in the direction of the autumnal equinox and lying in the equatorial plane of the Earth. K is a unit vector in the direction of the Earth's North Pole and J = K x I. This system is fixed in space, and the position vectors representing the sun, Polaris, and hopefully, preprogrammed orientations are to be given in this (I J K) system.

2.3.2 Vehicle System at Launch

A second reference coordinate system to be used is a right-handed body vehicle system ($\hat{X}_o$ $\hat{Y}_o$ $\hat{Z}_o$). $\hat{X}_o$ is a unit vector along the vehicle's roll axis, $\hat{Y}_o$ is a unit vector along the vehicle's pitch axis, and $\hat{Z}_o = \hat{X}_o \times \hat{Y}_o$ where $\hat{Z}_o$ is along the vehicle's yaw axis. All attitude systems on board the vehicle output with respect to this ($\hat{X}_o$ $\hat{Y}_o$ $\hat{Z}_o$) system. Also, the orientation of each on-board probe is initially referenced to the ($\hat{X}_o$ $\hat{Y}_o$ $\hat{Z}_o$) system.

2.3.3 Local Launcher System

A third coordinate system to be used is the local launcher vertical, East, and North system ($\hat{V}$ $\hat{E}$ $\hat{N}$). $\hat{N}$ is a unit vector tangent to the meridian circle through the launcher and pointing to true North, $\hat{E}$ is a unit vector tangent to the circle of latitude through the launcher and pointing East of North, and $\hat{V} = \hat{E} \times \hat{N}$. Since the gyro system is uncaged in the launcher whose position is given in this ($\hat{V}$ $\hat{E}$ $\hat{N}$) system, reference to this coordinate system is required in order to provide a history of the vehicle flight from lift-off to despin and the spacecarft's subsequent motions.

2.3.4 Ecliptic Inertial System

A fourth reference system is an ecliptic inertial (I_ε J_ε K_ε). The ecliptic plane determined by the orthonormal vectors I_ε and J_ε is defined as the plane of the Earth's orbit about the sun. Referring to the fixed inertial system, $I_\varepsilon = I$ and K_ε makes an angle ε with K. The angle of obliquity $\varepsilon = 23° 27' 8.2'' - .4684(t-1900)''$, t = the year in question and $J_\varepsilon = K_\varepsilon \times I_\varepsilon$.

2.3.5 Geocentric System

An intermediate reference system to be used is the geocentric system (i j k)

with i and j unit vectors in the equatorial plane of the Earth and i in the direction of the Greenwich Meridian Plane. The unit vector k is in the direction of the Earth's North Pole and k = i x j.

2.3.6 Uncaged Vehicle System

Another intermediate reference system to be used is the $(\hat{X}\ \hat{Y}\ \hat{Z})$ orthonormal system which positions the vehicle axis X with respect to the caged system $(\hat{X}_o\ \hat{Y}_o\ \hat{Z}_o)$. Using gyro yaw, pitch and roll information, a transformation matrix will be developed in Section 6 to transform this uncaged system to other required reference systems.

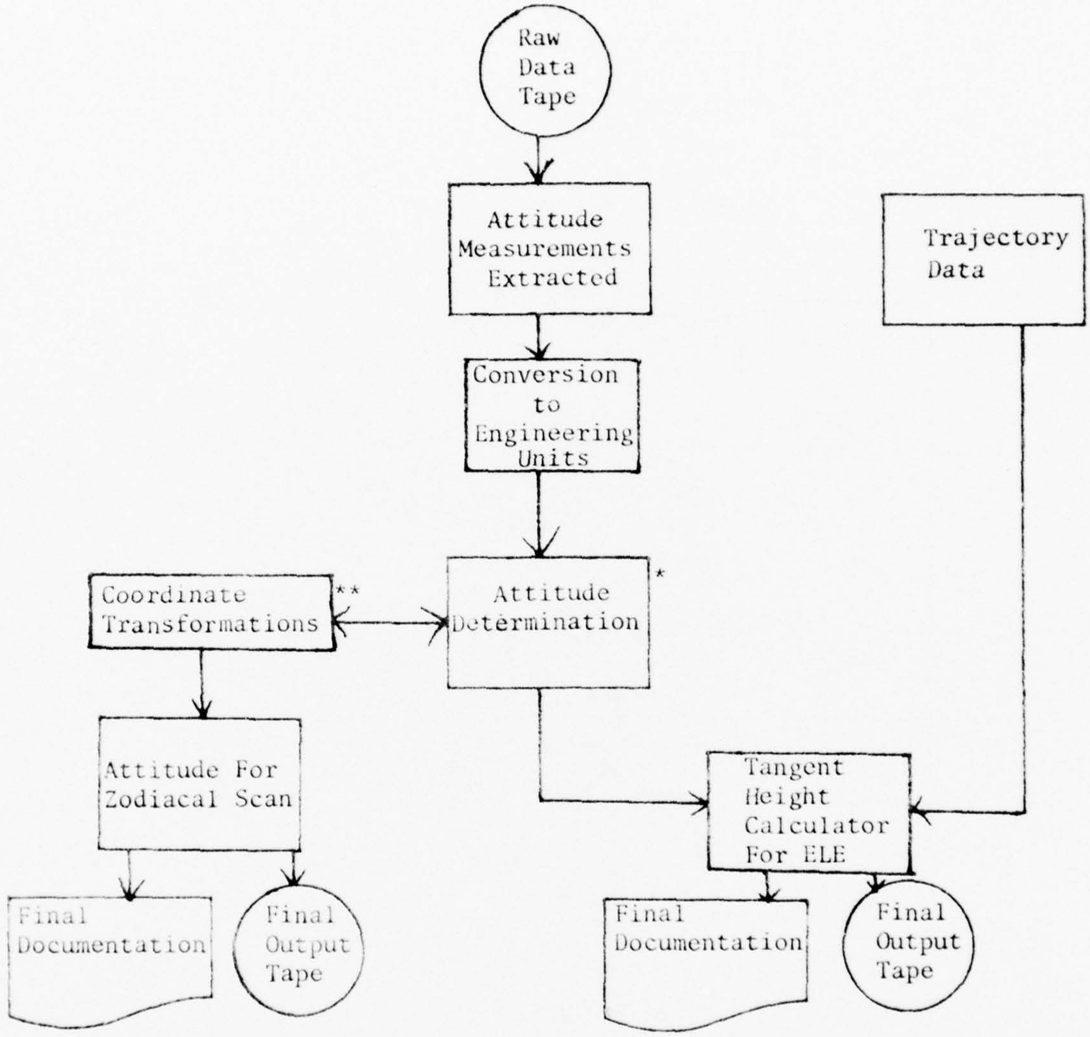
3. DATA FLOW FOR A35.191-4 (ELE/ZODIACAL)

The data flow commences with processing the digital tapes containing the attitude information into meaningful engineering units. This data is then corrected for any systematic error inherent to the attitude measuring systems.

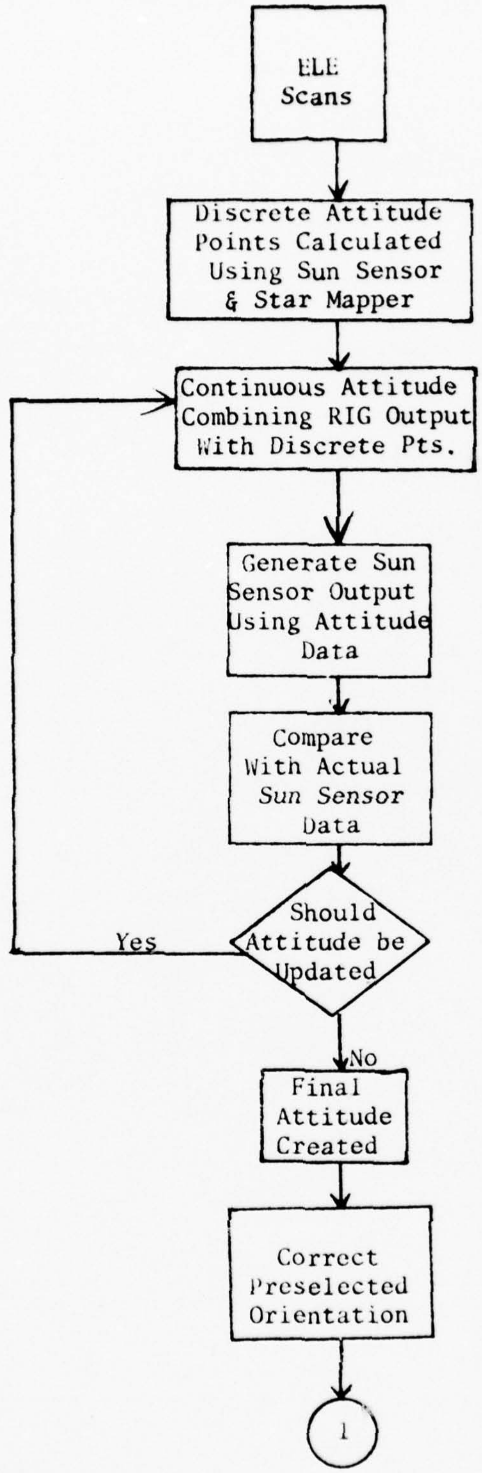
As previously discussed, the attitude for the vertical scans of the ELE will be calculated using the outputs of the sun sensor, star mapper, and RIG which is mounted on the scanning axis. This attitude determination will be constantly updated using the sun sensor information and, further, will allow the preselected orientation for the start of the ELE scans to be updated. This final attitude data will be combined with the ephemeris data to calculate tangent height.

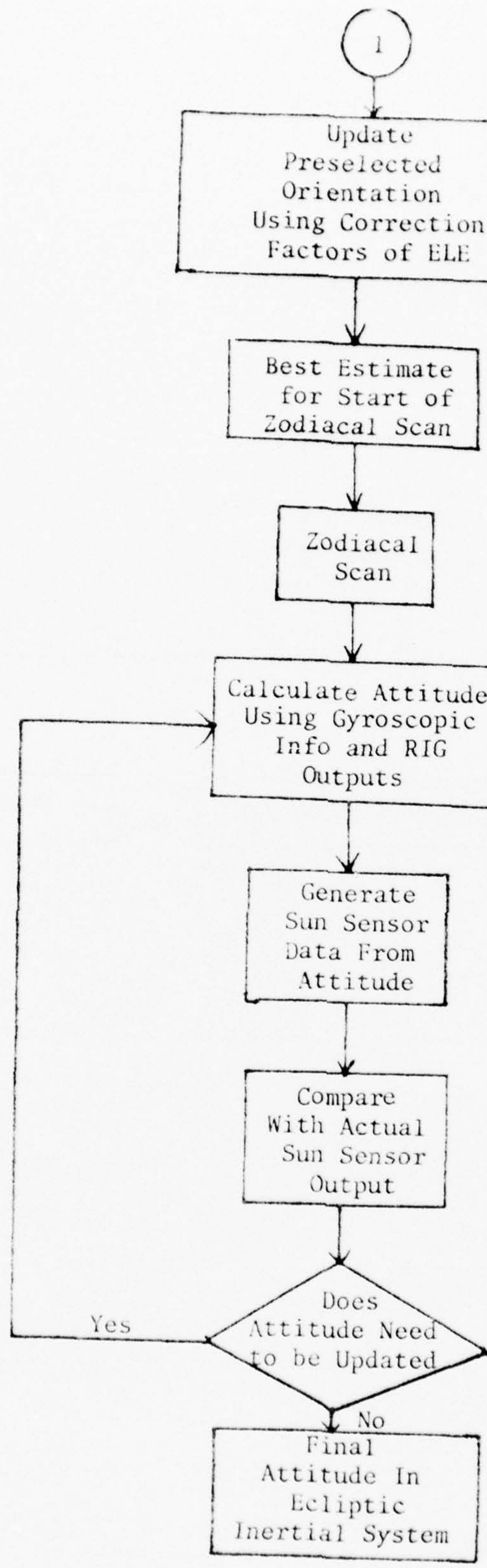
Since the vehicle and gyros are torqued to a preselected orientation for the start of the zodiacal scan and since the initial attitude for the zodiacal scan will be calculated using gyroscopic and RIG outputs whose zero reference will be this new orientation, it is essential for accurate attitude information during the zodiacal scan that this orientation be accurately known. According to a previous discussion, updates on this preselected position will be made by criteria established during the ELE scans, and the initial attitude information calculated for the zodiacal scan will be updated using the sun sensor output. The final attitude for the zodiacal scan will be represented in an inertial ecliptic system.

DATA FLOW FOR A35.191-4(ELE/ZODIACAL)

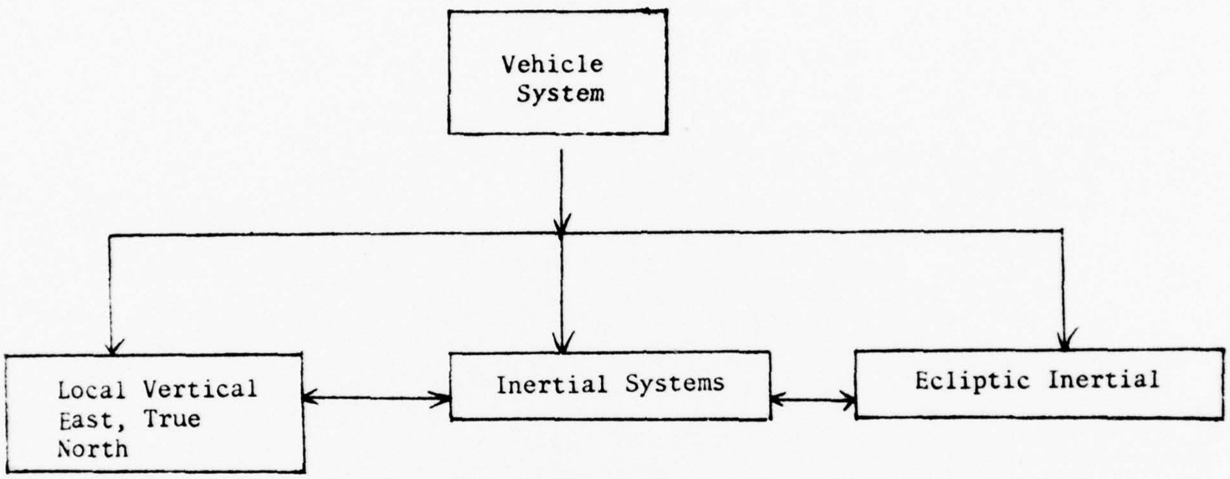


ATTITUDE DETERMINATION WITH UPDATES*




1
Update Preselected Orientation Using Correction Factors of ELE
Best Estimate for Start of Zodiacal Scan
Zodiacal Scan
Calculate Attitude Using Gyroscopic Info and RIG Outputs
Generate Sun Sensor Data From Attitude
Compare With Actual Sun Sensor Output
Does Attitude Need to be Updated
Yes
No
Final Attitude In Ecliptic Inertial System

COORDINATE TRANSFORMATIONS**



4. TANGENT HEIGHT CALCULATOR

The coordinate system to find tangent height is the fixed inertial system (I J K) where I is a unit vector in the direction of the vernal equinox and lying in the equatorial plane, K is a unit vector in the direction of the Earth's North Pole, and J = K x I.

The tangent height is calculated with reference to this (I J K) system by the orientation of the scanning axis $\overline{X}$ and vehicle altitude from the center of the Earth.

$\overline{X}$ is represented as

$$\overline{X} = \ell_x I + m_x J + n_x K$$

where ℓ_x, m_x, n_x are the attitude determined direction cosines of the scanning axis. The distance R_p from the center of the Earth to the vehicle is given by the expression

$$R_p = x_p I + y_p J + z_p K.$$

The equation for any point (x,y,z) on the oblate spheroidal Earth model is

$$\frac{x^2 + y^2}{b^2} + \frac{z^2}{c^2} = 1$$

where b and c, the equatorial and polar radii respectively, are

$$b = 6378.388 \text{ km.}$$
$$c = 6356.912 \text{ km.}$$

Then, for some point x_q, y_q, z_q on the Earth's surface, the vector from the center of the Earth to this point can be represented as

$$R_q = x_q I + y_q J + z_q K.$$

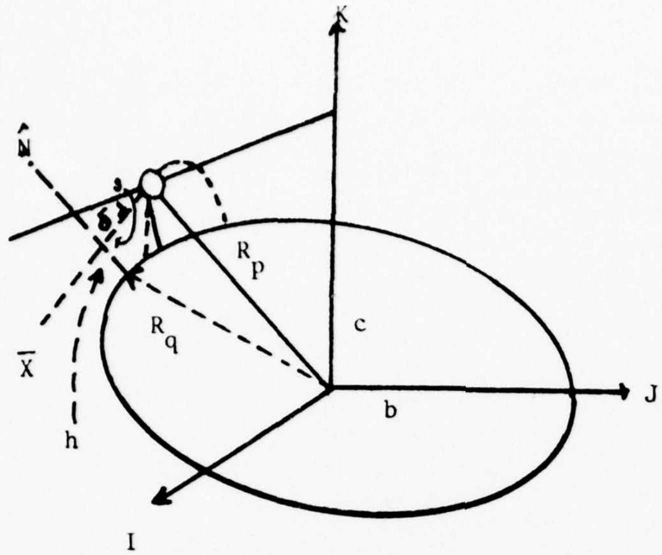


Figure 1

Since the gradient is normal to the surface $\phi(x,y,z)$ = constant through each point on the surface*, a normal $\hat{N}$ to the spheroidal model at x_q, y_q, z_q and to the scanning axis is

$$\hat{N} = \frac{I\dfrac{x_q}{b^2} + J\dfrac{y_q}{b^2} + K\dfrac{z_q}{c^2}}{\sqrt{\dfrac{x_q^{\,2} + y_q^{\,2}}{b^4} + \dfrac{z_q^{\,2}}{c^4}}} .$$

An attitude dependent expression for $\hat{N}$ can be defined as

$$\hat{N} = \overline{X} \times \frac{(R_p \times \overline{X})}{|R_p \times \overline{X}|}$$

since $\hat{N}$ is perpendicular to $\overline{X}$ and in a plane parallel to the plane of R_p and $\overline{X}$.**

*Phillips, Vector Analysis, pg. 35.

**Phillips, Vector Analysis, pg. 17.

Summing the vectors in Figure 1, we get

$$R_p + s\overline{X} - h\hat{N} - R_q = 0 \, . \tag{4-1}$$

Since $\overline{X}$ is perpendicular to $\hat{N}$,

$$\overline{X} \cdot \hat{N} = 0 \, .$$

Taking the scalar product of (4-1) with $\overline{X}$ and $\hat{N}$ respectively, we find that

$$s = \overline{X} \cdot (R_q - R_p)$$

$$h = \hat{N} \cdot (R_p - R_q) \, .$$

Given the attitude of $\overline{X}$ and the position vector R_p in the fixed system, the distance s in the $\overline{X}$ direction from the rocket in trajectory to $\hat{N}$ and the tangent height h are completely determined and can be rewritten as

$$s = \frac{\dfrac{(b^2-c^2)\; n_x\; [n_x(R_p \cdot \overline{X}) - z_p]}{\sqrt{|R_p|^2 - (R_p \cdot \overline{X})^2}}}{\sqrt{b^2 - \dfrac{(b^2-c^2)\; [z_p - n_x(R_p \cdot \overline{X})]^2}{|R_p|^2 - (R_p \cdot \overline{X})^2}}} - (R_p \cdot \overline{X})$$

$$h = \sqrt{|R_p|^2 - (R_p \cdot \overline{X})^2} - \sqrt{b^2 - \frac{(b^2-c^2)\; [z_p - n_x(R_p \cdot \overline{X})]^2}{|R_p|^2 - (R_p \cdot \overline{X})^2}} \, .$$

Let the direction cosines of $\overline{X}$ when the distance s=0 be given by ℓ_{x_o}, m_{x_o}, n_{x_o} and define

$$\overline{X}_o = \ell_{x_o} I + m_{x_o} J + n_{x_o} K \, ,$$

then the scan angle δ (Fig. 1) for any other direction $\overline{X}$ is

$$\delta = \cos^{-1}(\overline{X} \cdot \overline{X}_o) = \cos^{-1}(\ell_{x_o}\ell_x + m_{x_o}m_x + n_{x_o}n_x) .$$

5. STAR MAPPER COORDINATE TRANSFORMATIONS

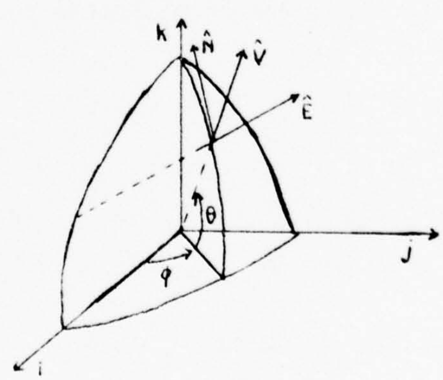
The mathematical and graphical relationships between the basic reference coordinate systems are presented here. Also, any intermediate results necessary to produce the aforementioned systems will be described.

5.1 Fixed Inertial System (I J K)

Since the history of vehicle flight is based upon the local $(\hat{V}\ \hat{E}\ \hat{N})$ system, a transformation to the fixed inertial (I J K) system must be made when providing sun and moon information. The relation of coordinate systems is necessitated by the fact that sun and moon data are provided in the (I J K) system. This transformation can be separated into two distinct phases, the first of which relates $(\hat{V}\ \hat{E}\ \hat{N})$ to the geocentric system (i j k) and the second of which relates the (i j k) system to the (I J K) system. Summarizing these transformations we have

$$\begin{bmatrix} \hat{V} \\ \hat{E} \\ \hat{N} \end{bmatrix} = A \begin{bmatrix} i \\ j \\ k \end{bmatrix} = AD \begin{bmatrix} I \\ J \\ K \end{bmatrix}.$$

The matrix A requires the latitude and longitude with respect to the Greenwich Meridian Plane of the position on the Earth's surface to which data is being referenced.



Θ : latitude

φ : longitude

Figure 2

The matrix D requires the coordinates of the sun, moon or any other body that is specified in the (I J K) system.

ϕ_s : apparent rt. ascension

V.E. : vernal equinox

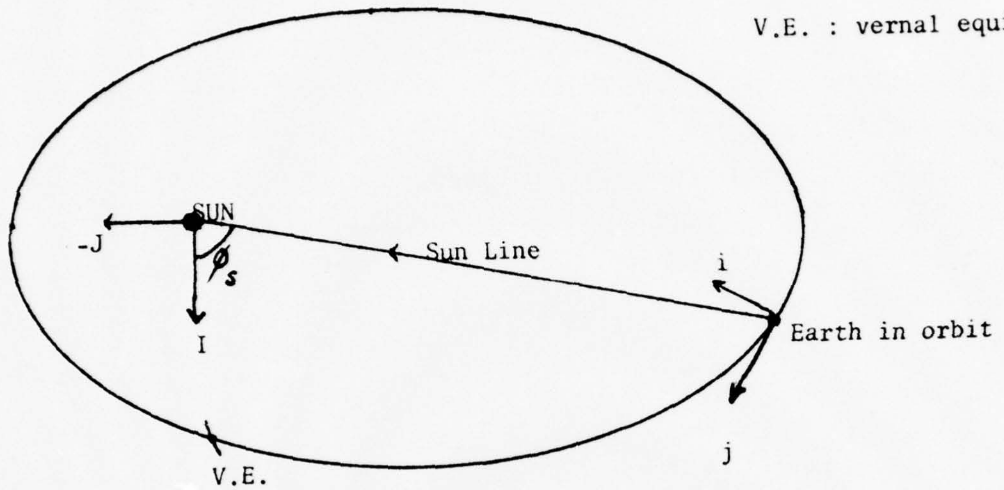


Figure 3

5.2 Vehicle System ($\hat{X}_o\ \hat{Y}_o\ \hat{Z}_o$)

Given the elevation and azimuth of the launcher in the ($\hat{V}\ \hat{E}\ \hat{N}$) system, we can relate the ($\hat{X}_o\ \hat{Y}_o\ \hat{Z}_o$) to the local system by the expression

$$\begin{bmatrix} \hat{X}_o \\ \hat{Y}_o \\ \hat{Z}_o \end{bmatrix} = B \begin{bmatrix} \hat{V} \\ \hat{E} \\ \hat{N} \end{bmatrix}$$

where the B matrix requires the launcher settings.

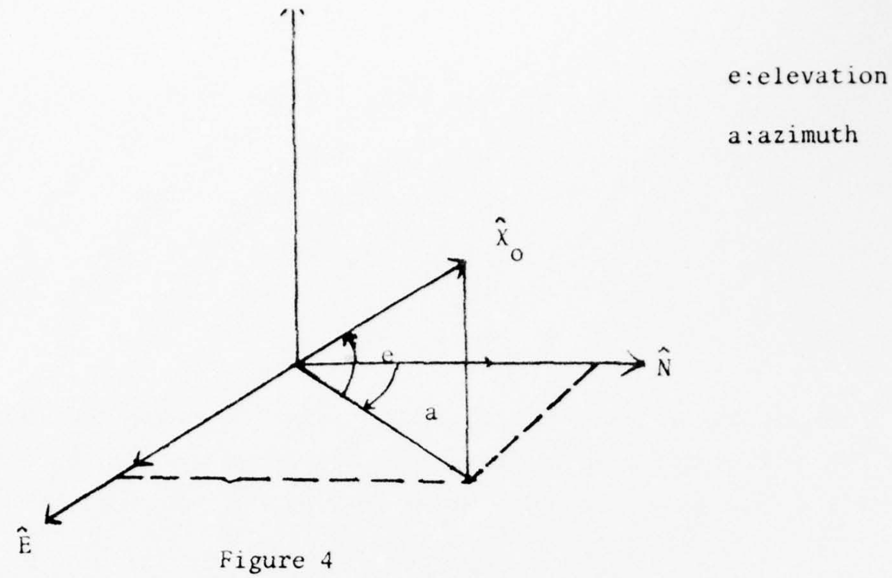


Figure 4

The following C matrix requires the gyro yaw, pitch and roll information during the vehicle lifetime. The $(\hat{X}\ \hat{Y}\ \hat{Z})$ orthonormal system positions the vehicle axis X with respect to the caged system $(\hat{X}_o\ \hat{Y}_o\ \hat{Z}_o)$.

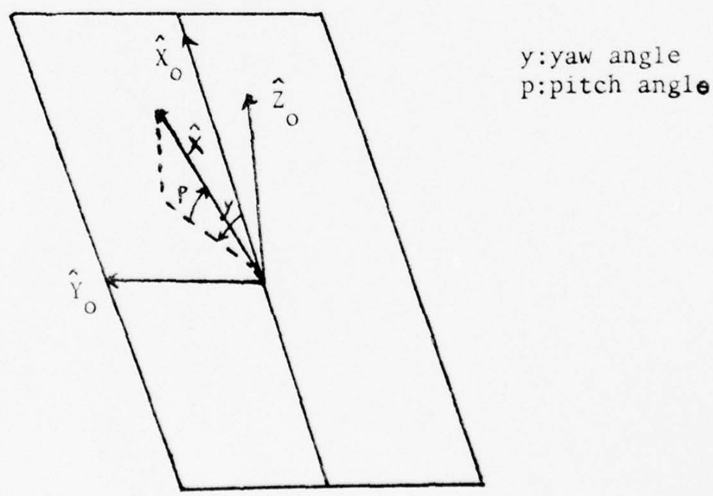


Figure 5

This transformation is expressed in matrix form by

$$\begin{bmatrix}\hat{X}\\ \hat{Y}\\ \hat{Z}\end{bmatrix} = C\begin{bmatrix}\hat{X}_o\\ \hat{Y}_o\\ \hat{Z}_o\end{bmatrix} .$$

To determine the attitude of any probe P, we must reference the orientation of the unit vector $\hat{P}$ in the direction of probe P to the $(\hat{X}_o\ \hat{Y}_o\ \hat{Z}_o)$ system. The matrix G determines the orientation of P in the intermediate $(\hat{X}\ \hat{Y}\ \hat{Z})$ system, i.e.,

$$\hat{P} = G\begin{bmatrix}\hat{X}\\ \hat{Y}\\ \hat{Z}\end{bmatrix} = GC\begin{bmatrix}\hat{X}_o\\ \hat{Y}_o\\ \hat{Z}_o\end{bmatrix} .$$

$$\left.\begin{matrix}\lambda\\ \\ \mu\end{matrix}\right\}\ \begin{matrix}\text{define probe position}\\ \text{with respect to the vehicle}\\ \text{axis X and Y reference.}\end{matrix}$$

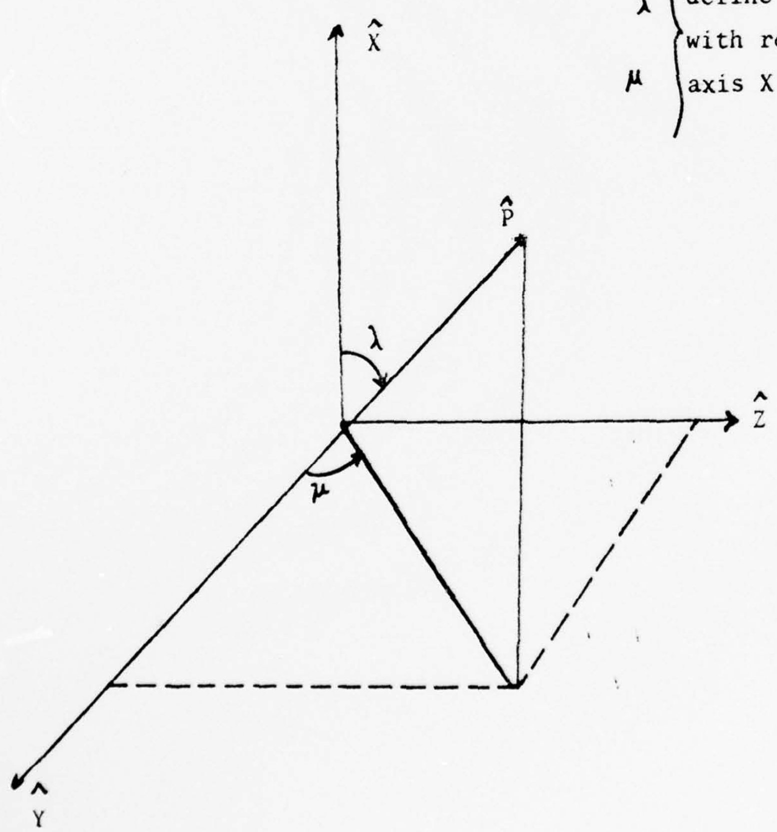


Figure 6

5.3 Local Vertical, East, North System $(\hat{V}\ \hat{E}\ \hat{N})$

The position of the vehicle axis in the launcher is given in the local $(\hat{V}\ \hat{E}\ \hat{N})$ system. This caged position serves as a starting point from which to provide a history of the vehicle flight from lift-off to despin. Utilizing the transformation matrices already discussed, we can now specify any probe P in the (I J K) system; i.e.,

$$\hat{P} = GCBAD \begin{bmatrix} I \\ J \\ K \end{bmatrix} .$$

Should the $(\hat{V}\ \hat{E}\ \hat{N})$ system be desired as the reference system for sun and moon data, the (I J K) system takes the form

$$\begin{bmatrix} I \\ J \\ K \end{bmatrix} = (AD)^T \begin{bmatrix} \hat{V} \\ \hat{E} \\ \hat{N} \end{bmatrix}$$

where $(AD)^T$ is the transpose of the matrix multiplication AD.

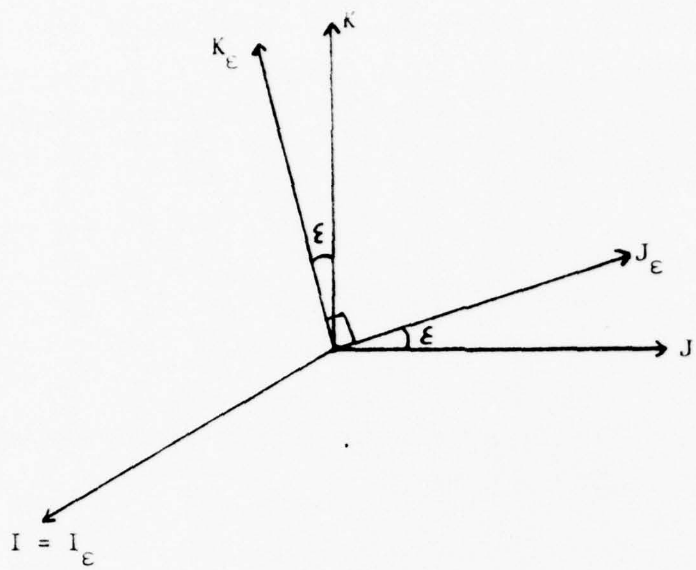
By the above transformations, we can now provide attitude for any probe P either with respect to the (I J K) system or the local $(\hat{V}\ \hat{E}\ \hat{N})$ system.

5.4 *Ecliptic Inertial System* $(I_\varepsilon\ J_\varepsilon\ K_\varepsilon)$

In order to reference data to the $(I_\varepsilon\ J_\varepsilon\ K_\varepsilon)$ system, we first express the (I J K) in the form

$$\begin{bmatrix} I \\ J \\ K \end{bmatrix} = F \begin{bmatrix} I_\varepsilon \\ J_\varepsilon \\ K_\varepsilon \end{bmatrix}$$

where F is the matrix which relates the base vectors of the (I J K) system to the base vectors of ecliptic system.



ε : angle of obliquity

$J_\varepsilon = K_\varepsilon \times I_\varepsilon$ (and J_ε is not in the plane of I and J)

Figure 7

Now, any probe P can now be referenced to the $(I_\varepsilon\ J_\varepsilon\ K_\varepsilon)$ system; i.e.,

$$\hat{P} = \text{GCBADF} \begin{bmatrix} I_\varepsilon \\ J_\varepsilon \\ K_\varepsilon \end{bmatrix}$$

This $(I_\varepsilon\ J_\varepsilon\ K_\varepsilon)$ system has been selected by the experimenter to be the reference system for the last portion of the flight of A35.191-4 during which the zodiacal scan occurs.

A figure relating the earth's orbital motion on the celestial sphere to the ecliptic follows.

A figure relating the earth's orbital motion on the celestial sphere to the ecliptic follows.

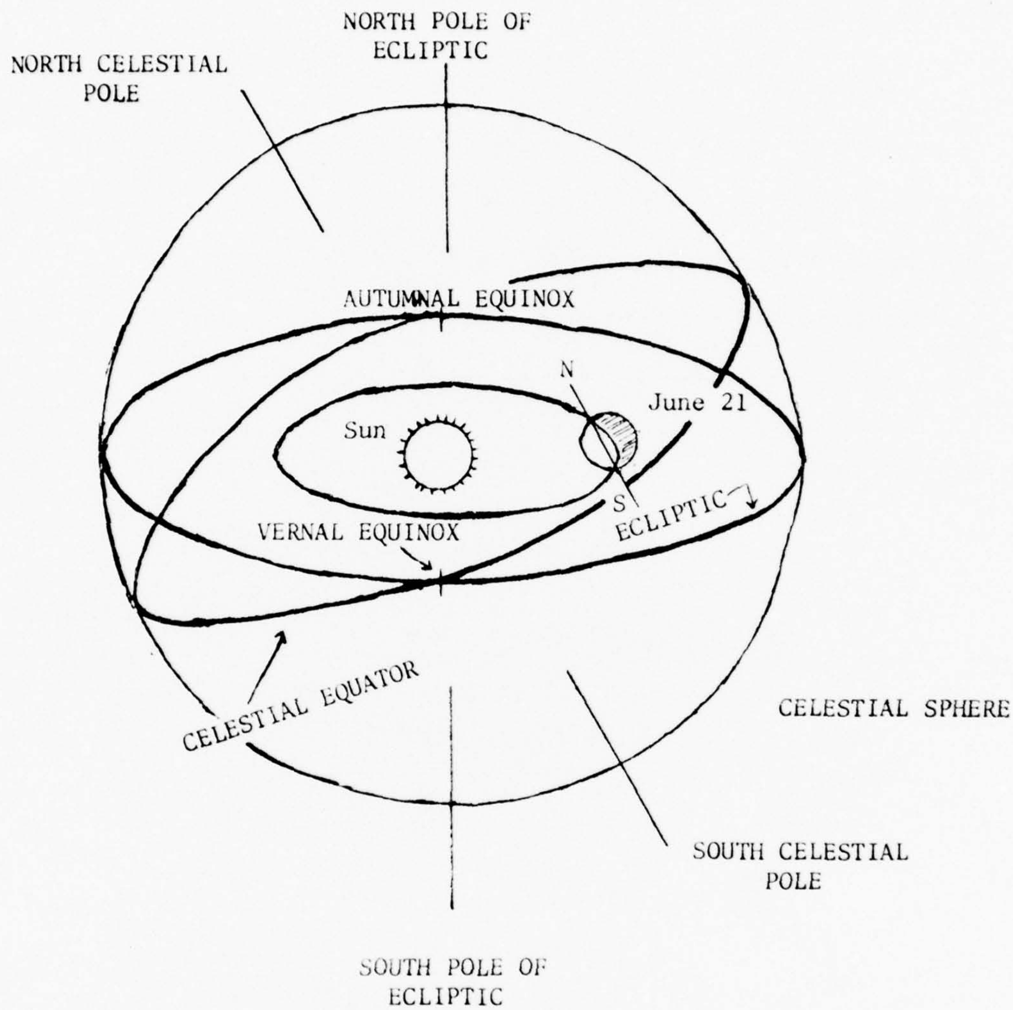


Figure 8

The mathematical expressions for each of the transformation matrices A, B, C, D, F, G follow in Section 6.

6. COORDINATE TRANSFORMATION MATRICES

The transformation matrices to be developed relate the following coordinate systems:

1) local system - $(\hat{V}\ \hat{E}\ \hat{N})$
2) geocentric system - $(i\ j\ k)$
3) fixed inertial system - $(I\ J\ K)$
4) caged vehicle system - $(\hat{X}_o\ \hat{Y}_o\ \hat{Z}_o)$
5) uncaged vehicle system - $(\hat{X}\ \hat{Y}\ \hat{Z})$
6) ecliptic inertial system - $(I_\varepsilon\ J_\varepsilon\ K_\varepsilon)$

From Figure 2

$$\hat{V} = i \cos\theta \cos\phi + j \cos\theta \sin\phi + k \sin\theta$$

$$\hat{E} = \frac{1}{\cos\theta} \frac{\partial \hat{V}}{\partial \phi} = -i \sin\phi + j \cos\phi$$

$$\hat{N} = \frac{\partial \hat{V}}{\partial \theta} = -i \sin\theta \cos\phi - j \sin\theta \sin\phi + k \cos\theta \ .$$

Since

$$\begin{bmatrix} \hat{V} \\ \hat{E} \\ \hat{N} \end{bmatrix} = A \begin{bmatrix} i \\ j \\ k \end{bmatrix} ,$$

then,

$$A = \begin{bmatrix} \cos\theta \cos\phi & \cos\theta \sin\phi & \sin\theta \\ -\sin\phi & \cos\phi & 0 \\ -\sin\theta \cos\phi & -\sin\theta \sin\phi & \cos\theta \end{bmatrix} .$$

For the caged vehicle system, $\hat{X}_o$ is along the vehicle's roll axis, $\hat{Y}_o$ is along the pitch axis, and $\hat{Z}_o$ is along the yaw axis with $\hat{X}_o = \hat{Y}_o \times \hat{Z}_o$.

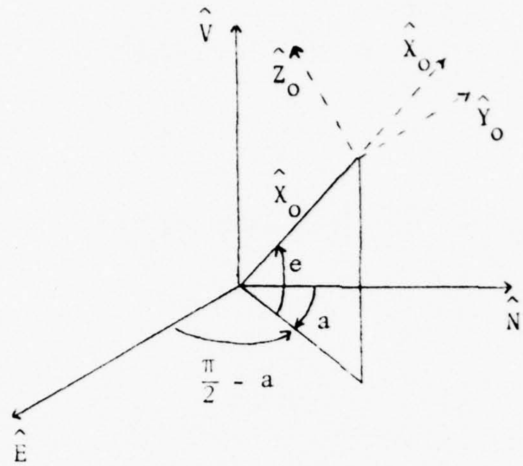


Figure 9

From the above figure

$$\hat{X}_o = \hat{V} \sin e + \hat{E} \cos e \cos(\frac{\pi}{2} - a) + \hat{N} \cos e \sin(\frac{\pi}{2} - a)$$

or

$$\hat{X}_o = \hat{V} \sin e + \hat{E} \cos e \sin a + \hat{N} \cos e \cos a$$

$$\hat{Y}_o = \frac{-1}{\cos e} \frac{\partial \hat{X}_o}{\partial a} = -\hat{E} \cos a + \hat{N} \sin a$$

$$Z_o = \frac{\partial X_o}{\partial e} = \hat{V} \cos e - \hat{E} \sin e \sin a - \hat{N} \sin e \cos a$$

Since

$$\begin{bmatrix} \hat{X}_o \\ \hat{Y}_o \\ \hat{Z}_o \end{bmatrix} = B \begin{bmatrix} \hat{V} \\ \hat{E} \\ \hat{N} \end{bmatrix} ,$$

then the matrix B is defined as

$$B = \begin{bmatrix} \text{sine} & \text{cose sina} & \text{cose cosa} \\ 0 & -\text{cosa} & +\text{sina} \\ \text{cose} & -\text{sine sina} & -\text{sine cosa} \end{bmatrix} .$$

From Figure 5 and [1]

$$\begin{aligned} \hat{X} &= \hat{X}_o \text{ cosp cosy} + \hat{Y}_o \text{ cosp siny} + \hat{Z}_o \text{ sinp} \\ \hat{Y}_1 &= \frac{1}{\text{cosp}} \frac{\partial \hat{X}}{\partial y} \\ \hat{Z}_1 &= \frac{\partial \hat{X}}{\partial p} \end{aligned} \tag{6-1}$$

With the roll angle r defined as the vehicle roll in the $\hat{Y}$, $\hat{Z}$ plane,

$$\begin{aligned} \hat{Y} &= \hat{Y}_1 \text{ cosr} + \hat{Z}_1 \text{ sinr} \\ \hat{Z} &= -\hat{Y}_1 \text{ sinr} + \hat{Z}_1 \text{ cosr} \end{aligned} \tag{6-2}$$

where $\hat{Y}$ and $\hat{Z}$ are displayed below.

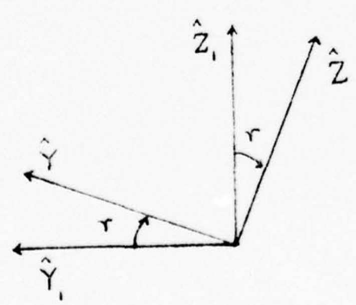


Figure 10

According to [1], $\hat{X}_1$, $\hat{Y}_1$, $\hat{Z}_1$ are linear combinations of the unit vectors in the direction of the gyro axes and the transformation matrix C in the expression

$$\begin{bmatrix} \hat{X} \\ \hat{Y} \\ \hat{Z} \end{bmatrix} = C \begin{bmatrix} \hat{X}_o \\ \hat{Y}_o \\ \hat{Z}_o \end{bmatrix}$$

is well defined.

To determine the matrix D in the expression

$$\begin{bmatrix} i \\ j \\ k \end{bmatrix} = D \begin{bmatrix} I \\ J \\ K \end{bmatrix}$$

we begin by letting I_o and J_o be mutually perpendicular unit vectors such that $K = I_o \times J_o = I \times J$.

In Figure 11, I_o is the sun line vector from the Earth to the sun and the apparent right ascension ϕ_s is the angle between the vectors I and $-I_o$.

---

[1] Roxborough et al, Procedures for the Determination of the Attitude of a Rocket from Gyroscopic Data, Boston College, Final Report prepared for Contract Number F19628-70-C-0017 (United States Air Force, 1972).

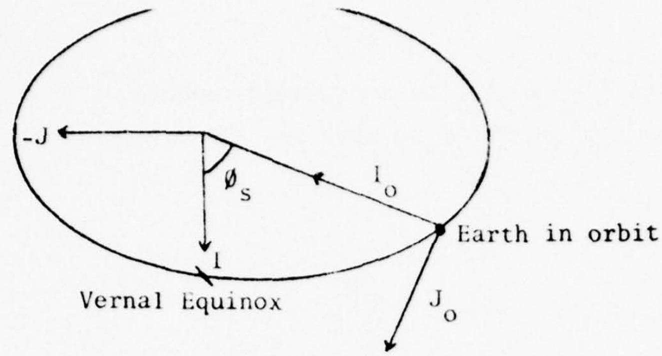


Figure 11

Since I_o and J_o are in the plane of I and J,

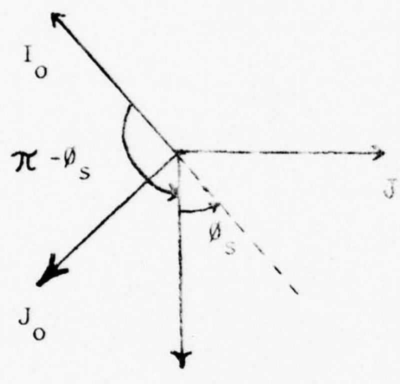


Figure 12

$$I_o = I\cos(\pi-\phi_s) - J\sin(\pi-\phi_s) = -I\cos\phi_s - J\sin\phi_s$$

$$J_o = I\cos(\pi-\phi_s - \tfrac{\pi}{2}) - J\sin(\pi-\phi_s - \tfrac{\pi}{2}) = I\cos(\tfrac{\pi}{2} - \phi_s) - J\sin(\tfrac{\pi}{2} - \phi_s) \tag{6-3}$$

or

$$J_o = I\sin\phi_s - J\cos\phi_s \ .$$

The geocentric system involves a translation of the I_o J_o K system

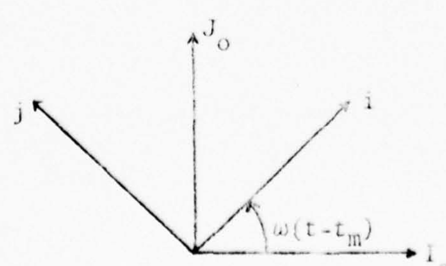


Figure 13

and

$$i = I_o \cos[\omega(t-t_m)] + J_o \sin[\omega(t-t_m)]$$

$$j = I_o \cos[\omega(t-t_m) + \tfrac{\pi}{2}] + J_o \sin[\omega(t-t_m) + \tfrac{\pi}{2}] \qquad (6\text{-}4$$

$$= -I_o \sin[\omega(t-t_m)] + J_o \cos[\omega(t-t_m)]$$

$$k = K\ .$$

Incorporating (6-3) with (6-4), we find that

$$i = -I\,[\cos\phi_s \cos[\omega(t-t_m)] - \sin\phi_s \sin[\omega(t-t_m)]]$$

$$- J\,[\sin\phi_s \cos[\omega(t-t_m)] + \cos\phi_s \sin[\omega(t-t_m)]]$$

$$j = I\,[\cos\phi_s \sin[\omega(t-t_m)] + \sin\phi_s \cos[\omega(t-t_m)]]$$

$$+ J\,[\sin\phi_s \sin[\omega(t-t_m)] - \cos\phi_s \cos[\omega(t-t_m)]]$$

$$k = K.$$

After simplifying the above equations,

$$i = -I \cos[\phi_s + \omega(t-t_m)] - J \sin[\phi_s + \omega(t-t_m)]$$

$$j = I \sin[\phi_s + \omega(t-t_m)] - J \cos[\phi_s + \omega(t-t_m)]$$

$$k = K.$$

and

$$D = \begin{bmatrix} -\cos[\phi_s + \omega(t-t_m)] & -\sin[\phi_s + \omega(t-t_m)] & 0 \\ \sin[\phi_s + \omega(t-t_m)] & -\cos[\phi_s + \omega(t-t_m)] & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

From Figure 7 with the angle of obliquity ε as defined in Section 2.3.4,

$$J_\varepsilon = J\cos\varepsilon + K\sin\varepsilon$$
$$K_\varepsilon = J\cos\left(\varepsilon + \frac{\pi}{2}\right) + K\sin\left(\varepsilon + \frac{\pi}{2}\right) = -J\sin\varepsilon + K\cos\varepsilon \quad . \tag{6-5}$$

After solving (6-5) for J and K, the transformation matrix F in the expression

$$\begin{bmatrix} I \\ J \\ K \end{bmatrix} = F \begin{bmatrix} I_\varepsilon \\ J_\varepsilon \\ K_\varepsilon \end{bmatrix}$$

can be expressed as

$$F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\varepsilon & -\sin\varepsilon \\ 0 & \sin\varepsilon & \cos\varepsilon \end{bmatrix} \quad .$$

To determine the orientation of a probe $\hat{P}$ in the uncaged vehicle system $(\hat{X}\ \hat{Y}\ \hat{Z})$, let

$$\hat{P} = G \begin{bmatrix} \hat{X} \\ \hat{Y} \\ \hat{Z} \end{bmatrix}$$

where G is a 1 by 3 row matrix and

$$\hat{P} = \alpha\hat{X} + \beta\hat{Y} + \gamma\hat{Z} \quad .$$

From Figure 6

$$\alpha = \cos\lambda$$
$$\beta = \sin\lambda\cos\mu$$
$$\gamma = \sin\lambda\sin\mu$$

which implies that

$$G = [\cos\lambda \quad \sin\lambda\cos\mu \quad \sin\lambda\sin\lambda] \quad .$$

7. SOFTWARE MODULES

A functional description, user's guide, and flow of the software modules developed for the A35.191-4 coordinate transformations will be presented. The coordinate systems and their codes are summarized in Table 1.

SYSTEM	DESCRIPTION	CODE
$\hat{X}_o, \hat{Y}_o, \hat{Z}_o$	at launch or caged	1
$\hat{V}, \hat{E}, \hat{N}$	local	2
i, j, k	geocentric	3
I, J, K	fixed inertial	4
$I_\varepsilon, J_\varepsilon, K_\varepsilon$	ecliptic	5

Table 1

7.1 Functional Description

The probe vector $\hat{P}$ can be expressed as

$$\hat{P} = G \begin{bmatrix} \hat{X} \\ \hat{Y} \\ \hat{Z} \end{bmatrix} = GC \begin{bmatrix} \hat{X}_o \\ \hat{Y}_o \\ \hat{Z}_o \end{bmatrix} = GCB \begin{bmatrix} \hat{V} \\ \hat{E} \\ \hat{N} \end{bmatrix} = GCBA \begin{bmatrix} i \\ j \\ k \end{bmatrix} = GCBAD \begin{bmatrix} I \\ J \\ K \end{bmatrix} = GCBADF \begin{bmatrix} I_\varepsilon \\ J_\varepsilon \\ K_\varepsilon \end{bmatrix}$$

where the $(\hat{X}\ \hat{Y}\ \hat{Z})$ system is a vehicle system in the uncaged position and the G, C, B, A, D, F matrices are transformation matrices.

Since we are assuming an attitude point for the probe vector $\hat{P}$ is known, the matrices G and C need not be computed.

If we are given the attitude of $\hat{P}$ in system code 1 and want the attitude in system code 3, the 1 x 3 array GC will be multiplied on the right by the 3 x 3 array BA. This will provide the attitude in the i, j, k system.

If we are given the attitude of $\hat{P}$ in system code 5 and want the attitude in system code 2, the 1 x 3 array GCBADF will be multiplied on the right by $(ADF)^T$, i.e.,

$$(GCBADF)\ (ADF)^T = GCBADFF^T D^T A^T = GCB$$

which yields the attitude in the requested $(\hat{V}\ \hat{E}\ \hat{N})$ system.

Given the input system code to be L1 and the output system code L2, a summary of the matrix multiplication follows:

1. For L1 < L2, successive multiplication of the transformation matrices occurs on the right.
2. For L1 > L2, the product of the successive transformation matrices when going from the L2 system to the L1 system is computed. Let this product matrix be called H, then the input attitude array is multiplied on the right by H^T.

7.2 User's Guide

The package consisting of

Subroutine INIT1 (E, AZ, TH, PH)
Subroutine INIT2 (PHIS, TM, T)
Subroutine COORD (L1, L2)

will convert the attitude of a probe from the L1 system code to the L2 system code (Table 1). The argument parameters are described below:

i) E = elevation of rocket axis when uncaged (degrees)
AZ = azimuth of rocket axis when uncaged (degrees)
TH = latitude (degrees) at launch
PH = longitude (degrees) pos. east - at launch

These quantities are to be defined in the routine calling INITI. INITI should be called only once.

ii) PHIS = right ascension (degrees)

TM = ephemeris transit time (total seconds GMT)

T = time of attitude point (total seconds GMT)

PHIS and TM need only be defined initially, but T must be updated before calling INIT2.

iii) L1 = input attitude system code

L2 = output attitude system code

ATTIN(1), ATTIN(2), ATTIN(3) direction cosines of the probe vector in the L1 system (radians). These quantities will pass from the calling routine to COORD via unlabelled COMMON.

ATTOUT(1), ATTOUT(2), ATTOUT(3) direction cosines of trh probe vector in the L2 system (radians). They form part of the unlabelled COMMON block.

L1, L2 and ATTIN array must be defined in the calling routine and be associated with time T. The ATTOUT array is the requested attitude of the probe vector.

iv) COMMON blocks

a) COMMON/DATA1/A(2,2),B(3,3),F(3,3)
to be placed in subroutines INIT1 and the calling routine.
A, B, F, are transformation arrays.

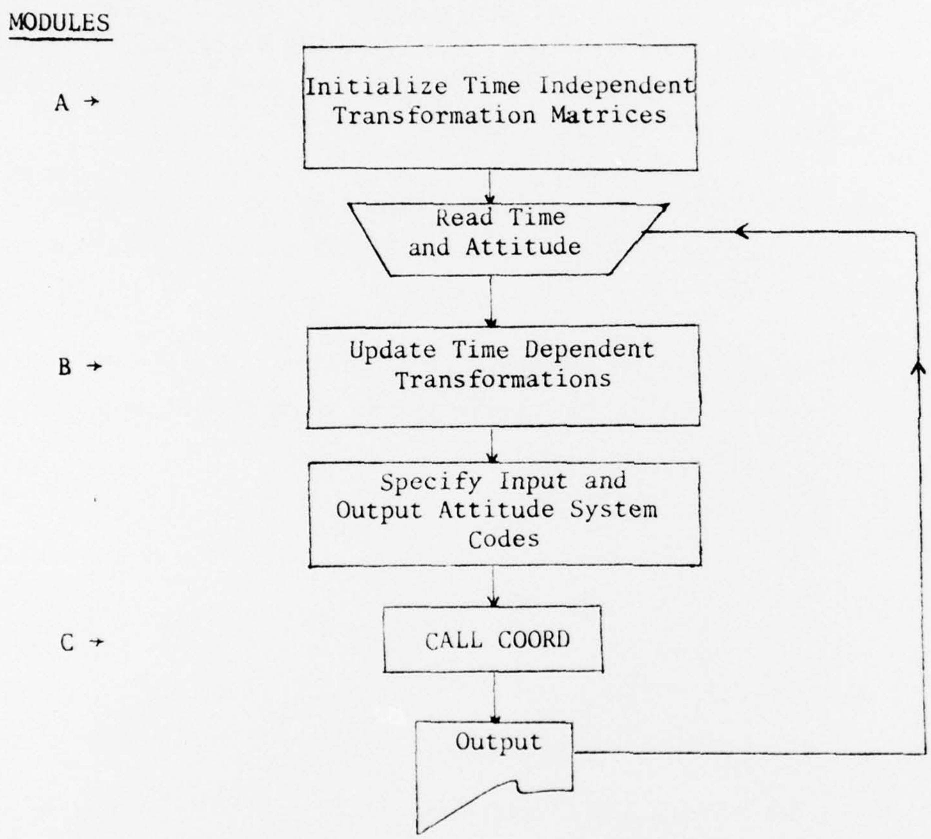
b) COMMON/DATA2/D(3,3)
to be placed in a subroutine INIT2 and the calling routine.
D is a transformation array.

c) COMMON H(3,3,4),ATTIN(3),ATTOUT(3)
to be placed in subroutine COORD and the calling routine.

H(I,J,1) = B(I,J), H(I,J,2) = A(I,J), H(I,J,4) = F(I,J) and H(I,J,3) = D(I,J).

7.3 Software Flow

The software transformation routines involve three modules. The first of these initializes the time independent transformation matrices, the second updates time dependent transformations, and the third computes the direction cosines of the probe vector in the output system.



8. KALMAN FILTER

In order to apply the Kalman filter routines to the attitude determination problem for A35.191-4, a dynamical model for the payload portion of the flight is required. The usual relationships between the angular velocities of the body and the Euler angles describing the orientation of the body will be used. Therefore, the previously defined coordinate systems should not be confused with those presented in this section.

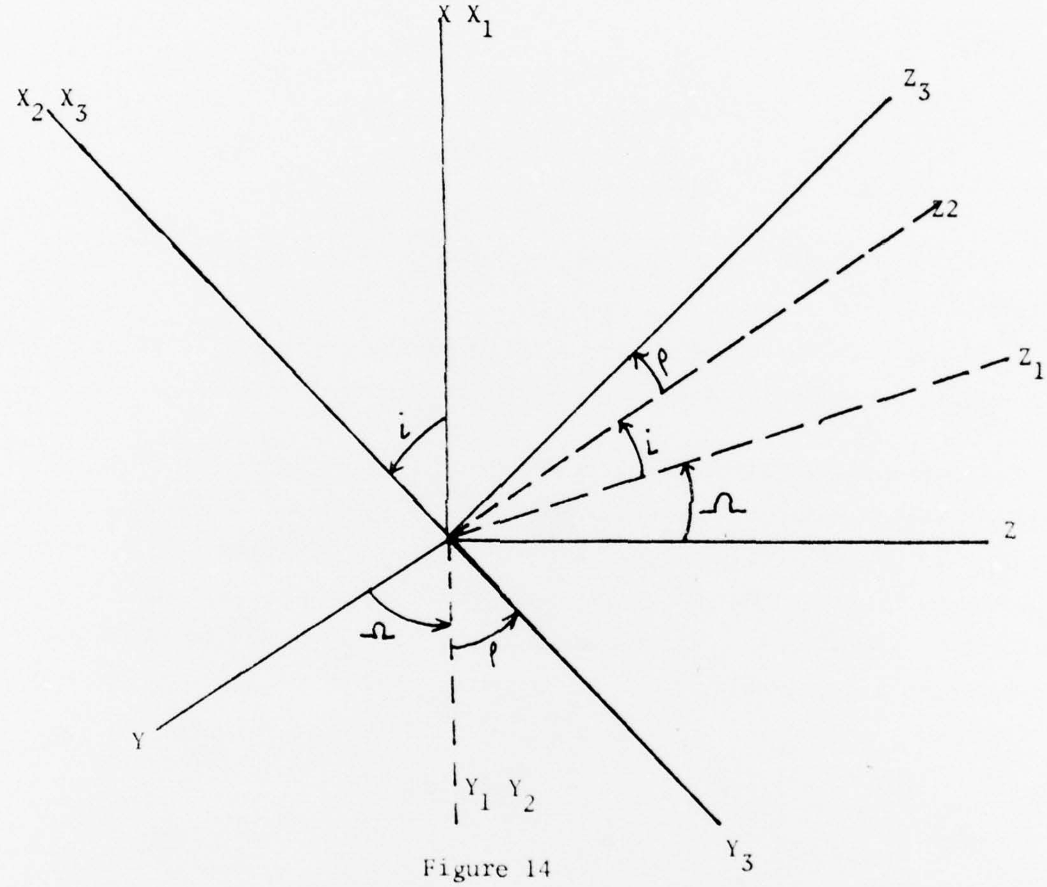


Figure 14

The X Y Z coordinate system is the inertial reference system used. The X_3 Y_3 Z_3 system is the body axes system. The other systems indicated illustrate intermediate phases in the transformation by successive rotations from the inertial system to the body axes system. In general, a vector (x, y, z) in the inertial system can be transformed to its representation (x_3, y_3, z_3) in the body axes system as follows:

$$\begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix} = R_1(\rho)\ R_2(i)\ R_1(\Omega) \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

where $R_j(\alpha)$ represents a right-handed rotation through the angle α about the j-axis. With the above definition of Euler angles, the dynamical relationship between angular velocities and Euler angles is:

$$\begin{aligned} \dot{\Omega} &= \frac{1}{\sin i}(\omega_y \sin\rho + \omega_z \cos\rho) \\ \dot{i} &= \omega_y \cos\rho - \omega_z \sin\rho \\ \dot{\rho} &= \omega_x - \frac{\cos i}{\sin i}(\omega_y \sin\rho + \omega_z \cos\rho) \end{aligned} \tag{8.1}$$

where $(\omega_x, \omega_y, \omega_z)$ is the angular velocity vector expressed in the principal axes system (coincident with body axes system in this case).

In the possible situation in equations (8.1) in which $\sin i \sim 0$, a new set of Euler angles will be used in order to avoid numerical difficulty. The new set is a longitude-latitude-spin angle approach to defining the Euler angles. In order to maintain the usual convention of taking the latitude δ to be positive in the direction toward positive z, the following relation between the vector's representations in the inertial system and the body axes system is defined:

$$\begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix} = R_1(\nu)\ R_2(-\delta)\ R_3(\lambda) \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

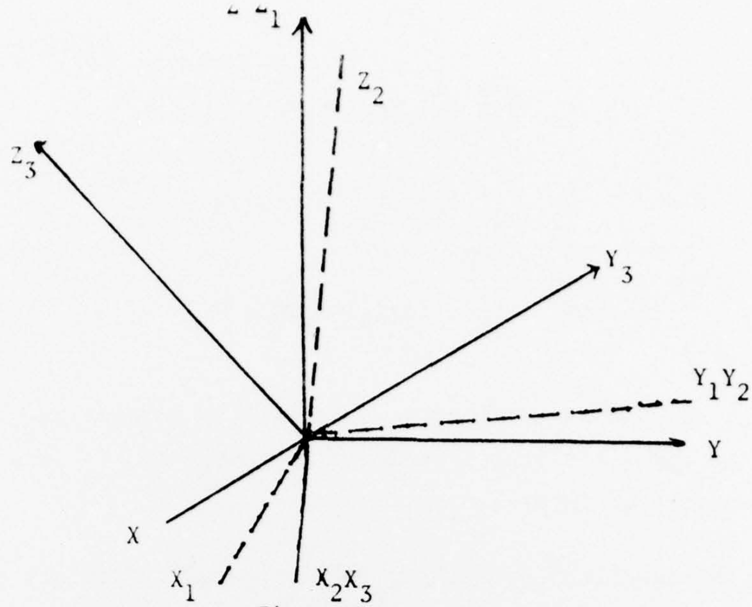


Figure 15

The relationship between the angular velocities and the Euler angles is:

$$
\begin{aligned}
\dot{\lambda} &= \frac{1}{\cos\delta}(\omega_y \sin\nu + \omega_z \cos\nu) \\
\dot{\delta} &= \omega_z \sin\nu - \omega_y \cos\nu \\
\dot{\nu} &= \omega_x - \frac{\sin\delta}{\cos\delta}(\omega_y \sin\nu + \omega_z \cos\nu) \ .
\end{aligned}
\tag{8.2}
$$

In the problem case when $\sin i \sim 0$, there will be no problem with equations (8.2) since $\cos\delta \sim 1$.

Solving for one set of Euler angles in terms of the other set is accomplished quite simply by manipulating certain terms of the common transformation matrix. When switching from one set to the other, the set of differential equations will be switched also.

The dynamical equations for the angular velocities are simply Euler's equations:

$$\dot{\omega}_x = \frac{1}{I_x} [(I_y - I_z) \omega_y \omega_z + M_x]$$

$$\dot{\omega}_y = \frac{1}{I_y} [(I_z - I_x) \omega_z \omega_x + M_y] \tag{8.3}$$

$$\dot{\omega}_z = \frac{1}{I_z} [(I_x - I_y) \omega_x \omega_y + M_z] .$$

(M_x, M_y, M_z) represents the torque which will be modeled as the nominal output of the thrusters plus a random component. I_x, I_y, and I_z are the principal moments of inertia.

Since the equations describing the dynamical system are non-linear, they will be linearized about a state reference vector. The partial derivatives of the righthand sides of equations (8.1), (8.2), and (8.3) are practically identical to the same equations programmed for a previous attitude determination problem.

The observations can be directly related to various elements of the state vector. The outputs of the rate gyros and the rate integrating gyros can be directly compared with the current estimates of the angular velocity vector $(\omega_x, \omega_y, \omega_z)$ since the body axes rates are directly sensed by these gyros. The angular sun and star data will be related to the Euler angles in the same manner as programmed for a previous attitude determination problem. If roll, pitch, and yaw displacements from gyro null positions need to be used, they will be related to the Euler angles by comparing the transformation matrix based on the sensor outputs with the transformation matrix formed from the current estimate of the Euler angles.

In applying the Kalman filter to the system of non-linear state equations, the standard linearization techniques are followed. The state equation is expanded in a Taylor series about $X = X_{ref}$, and all second- and higher-order terms are truncated. After subtracting the reference state equation, the state residuals equation remains:

$$\dot{x} = A(t)\, x + B(t)\, w \tag{8.4}$$

where $x = X - X_{ref}$, $A(t)$ is the matrix of partial derivatives of the right-hand sides of equations (8.1) or (8.2) and equations (8.3), $B(t)$ is chosen as a constant matrix for this problem, and $w(t)$ is the random component of the torque. In accordance with the theory of ordinary differential equations, a solution for equation (8.4) can be written in terms of the state transition matrix Φ:

$$x(t) = \Phi(t,t_o)\ x(t_o) + \int_{t_o}^{t} \Phi(t,\tau)\ B(\tau)\ w(\tau)\ d\tau$$

A mean solution can be written:

$$\overline{x}(t) = E[x(t)] = \Phi(t,t_o)\ \overline{x}(t_o) + \int_{t_o}^{t} \Phi(t,\tau)\ B(\tau)\ \overline{w}(\tau)\ d\tau\ .$$

If the following statistical assumptions are made:

$$E\ [w(t)] = 0$$

$$E\ [(x(t_o) - \overline{x}(t_o))\ (x(t_o) - \overline{x}(t_o))^T] = P(t_o)$$

$$E\ [(x(t_o) - \overline{x}(t_o))\ (w(t) - \overline{w}(t))^T] = 0$$

$$E\ [(w(t) - \overline{w}(t))\ (w(s) - \overline{w}(s))^T] = Q(t)\ \delta(t-s)$$

where $\delta(t-s)$ represents the Dirac delta function; then the variance solution can be written:

$$P(t) = E\ [(x(t) - \overline{x}(t))\ (x(t) - \overline{x}(t))^T]$$

$$= \Phi(t,t_o)\ P(t_o)\ \Phi^T(t,t_o) = \int_{t_o}^{t} \Phi(t,\tau)\ B(\tau)\ Q(\tau)\ B^T(\tau)\ \Phi^T(t,\tau)\ d\tau$$

If the variance solution is differentiated, the following differential equation for $P(t)$ is obtained:

$$\dot{P} = AP + PA^T + BQB^T$$

where the fact that $P = P^T$ was used. This equation is used to map the state covariance matrix P from one time to the next. In mapping P forward over a small interval from t_o to t, $P(t_o)$ is known; A is computed from the state reference vector, which is integrated simultaneously; B is a constant matrix; and Q is an input quantity representing the covariance of the state disturbance.

A similar linearization process involves the observation-state relational equation at time t_i,

$$Y(t_i) = G(X(t_i),t_i) + v_i \tag{8.5}$$

where Y is an m×1 vector-valued function, and v_i is an m×1 vector representing noise associated with the observation. A reference observation is computed using the equation,

$$Y_{ref}(t_i) = G(X_{ref}(t_i),t_i) \ .$$

In applying the Kalman filter, the partial derivatives of G are required. This matrix H is defined as:

$$H = \frac{\partial G}{\partial X}\bigg|_{X = X_{ref}} \ .$$

Equation (8.5) is expanded in a Taylor series about $X = X_{ref}$, and all second- and higher-order terms are truncated. After subtracting the reference observation-state relational equation, the observation residuals equation remains:

$$y = H(t)\ x + v$$

where

$$y = Y - Y_{ref} \ .$$

The Kalman filter is applied in this problem in accordance with the following general scheme:

a. Assume that at time t_i the following quantities are available:

$X_{ref}(t_i)$ state reference vector

$P(t_i) = P_i$ state covariance matrix

The matrix P_i gives a measure of the error in the best estimate of the state vector at time t_i. Assume also that P_i is based on q observations already processed.

b. At time t_k $(t_k > t_i)$, a new data record is processed. This data record contains new observations Y_{kj} at times t_{kj} respectively $(j = 1,2,\ldots,s)$. The covariance matrices $R_{kj} = E\,[v_{kj}\, v_{kj}^T]$ $(j = 1,2,\ldots,s)$ are also given.

c. The problem is to find a best estimate of the state vector $\hat{X}(t_k)$ and the associated covariance matrix P_k based on (q+s) observations.

d. Map forward $X_{ref}(t_i)$ to $X_{ref}(t_k)$ and $P(t_i)$ to $\overline{P}(t_k)$. Note that $\overline{P}(t_k) = \overline{P}_k$ is based only on q observations.

e. Map forward $X_{ref}(t_k)$ to $X_{ref}(t_{kj})$ $(j = 1,2,\ldots,s)$. Compute the corresponding state transition matrices $\Phi(t_{kj},t_k)$ $(j = 1,2,\ldots,s)$. Compute the reference observations $Y_{ref}(t_{kj}) = G(X_{ref}(t_{kj})t_{kj})$ and the corresponding partial derivatives $H(t_{kj}) = H_{kj}$ $(j = 1,2,\ldots,s)$. Find the observation residuals:

$$y_{kj} = Y_{kj} - Y_{ref}(t_{kj}) \quad (j = 1,2,\ldots,s) \ .$$

f. The following set of relationships holds:

$$\begin{aligned}
y_{k1} &= H_{k1}\, x(t_{k1}) + v_{k1} \\
y_{k2} &= H_{k2}\, x(t_{k2}) + v_{k2} \\
&\vdots \\
y_{ks} &= H_{ks}\, x(t_{ks}) + v_{ks}
\end{aligned}$$

Equivalently, this set can be written:

$$
\begin{aligned}
y_{k1} &= H_{k1}\ \Phi(t_{k1},t_k)\ x(t_k) + v_{k1} \\
y_{k2} &= H_{k2}\ \Phi(t_{k2},t_k)\ x(t_k) + v_{k2} \\
&\ \vdots \\
\underbrace{y_{ks}}_{y} &= \underbrace{H_{ks}\ \Phi(t_{ks},t_k)}_{H}\ \underbrace{x(t_k)}_{x(t_k)} + \underbrace{v_{ks}}_{v}
\end{aligned}
$$

g. The Kalman filter computes the best estimate of the state residuals vector $\hat{x}(t_k)$ and the state covariance matrix P_k.

$$\hat{x}(t_k) = K_k\ y$$

$$P(t_k) = (I - K_k H)\ \overline{P}_k$$

where

$$K_k = \overline{P}_k H^T\ [H\overline{P}_k H^T + R]^{-1}$$

$$R = E\ [v\ v^T]$$

and I is the identity matrix.

h. The best estimate of the state $\hat{X}(t_k)$ is simply:

$$\hat{X}(t_k) = X_{ref}(t_k) + \hat{x}(t_k) \quad .$$

i. In preparation for the next step, $X_{ref}(t_k)$ is redefined to be $\hat{X}(t_k)$; and the process continues with step (b).

9. ERROR ANALYSIS

An error analysis will be performed to determine optimum data flow and weighting criteria for individual probe outputs. This analysis will identify error sources, both systematic and random, define the magnitude and resultant contributions of the individual sources, and thus estimate the total errors. Included in this study will be expected random and bias errors caused by sensor alignment tolerances, timing uncertainties, ACS cross coupling errors, and random as well as systematic drifts. Non-random distributions will be assumed so that Root Mean Square (RMS) accuracy bands will be applicable.

This error analysis will allow us to evaluate the contributions of each error source to the final output and thus to determine which error sources can be ignored, how to minimize bias errors in the post flight data analysis and how best to perform timing synchronization.